LARGE SEPARATED SETS OF UNIT VECTORS IN BANACH SPACES OF CONTINUOUS FUNCTIONS

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A set A in a Banach space X is said to be $(1+)$-*separated* (resp. 2-*equilateral*) if $\|u - v\| > 1$ (resp. $\|u - v\| = 2$) for distinct $u, v \in A$.

We will discuss the problem whether a nonseparable $\mathcal{C}(K)$ space necessarily contains a $(1+)$-separated set of unit vectors whose cardinality equals to the density of $\mathcal{C}(K)$. We show that this is the case if the density is at most continuum. This improves a result of T. Kania and T. Kochanek.

Moreover, we show that for several classes of $\mathcal{C}(K)$ spaces it is even possible to find such a set which is 2-equilateral. This can not hold in full generality, since it was proven by P. Koszmider that it is undecidable in ZFC whether there always exists an uncountable 2-equilateral set in the unit sphere of a nonseparable $\mathcal{C}(K)$ space.

The results were achieved in collaboration with Marek Cúth and Benjamin Vejnar.

References

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